Homework # 10 for Math 123, Fall 2016

Due: November 15, in class

1. Brauer and Nohel: Section 4.3, page 154, # 7, # 8. Also do # 11 for these two problems.
2. Brauer and Nohel: Section 4.3, page 154, # 10
3. Brauer and Nohel: Section 4.3, page 154, # 12.
4. Brauer and Nohel: Section 4.3, page 159, # 17.
5. Find the critical points of each of the following systems of ODE, and classify them (asymptotically stable, stable, unstable).

   (a)

$$\begin{aligned} y_1' &= y_1 y_2 \\ y_2' &= 2(2 - y_2 - 2y_1) \end{aligned}$$

   (b)

$$\begin{aligned} y_1' &= 2y_1 y_2 - y_1^2 + 5y_1 \\ y_2' &= y_1^2 + y_2^2 - 6y_1 - 8y_2 \end{aligned}$$

6. Recall the Van der Pol equation, which we discussed in the third lecture:

$$y'' + \mu(y^2 - 1)y' + y = 0,$$

   where $\mu \in \mathbb{R}$ is a parameter. Formulate the Van der Pol equation as a linear system and find all of the critical points of the equation. Describe the nature of the critical points as a function of μ.

7. Brauer and Nohel: Section 4.5, page 177, # 4. Also write a sentence or two describing the stable and unstable trajectories.

Acknowledgement: the fifth and sixth problems of this homework set are based on problems from the textbook of Agarwal and O'Regan.